

MASS TRANSFER REGIMES IN REACTIONS
WITH A DISPERSED PHASE

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Equations are formulated which determine the movement of phases and the integral inter-phase mass transfer in isothermal two-phase reactors. Simple models of extraction which are not accompanied by a change of the specific volume of the phases and the dissolution of gas in the countercurrent columns and apparatuses with a horizontal flow of liquid are discussed. Some conclusions obtained differ considerably from the traditional ideas of the theory of solvent extraction. Mass transfer between the dispersion (continuous) medium and the dispersed (disperse) phase in the presence of relative movement of the phases are considered below. Such processes are realized in practice in various bubbling and atomizing devices (extraction or absorption countercurrent columns, some types of scrubbers, plate and other apparatuses with a horizontal flow of the continuous medium, etc.). Of primary interest is the macroscopic description of mass transfer; the quantities characterizing local mixing in the reactor and momentum and mass transfer between the dispersion medium and a single particle (drop or bubble) of the dispersed phase are assumed to be known functions of the macroscopic parameters.

1. Let there be in a system J components participating in mass transfer and in the chemical reactions accompanying this process, their weight concentrations in the dispersed and continuous phases being equal, respectively, to c_{1j}' and c_{2j}' ($j = 1, \dots, J$). Considering the material balance of these components, we write the corresponding equations for c_{1j}' and c_{2j}' in both phases:

$$\begin{aligned} \frac{\partial}{\partial t}(\rho c_{1j}') + \frac{\partial}{\partial \mathbf{r}}(\rho \mathbf{w} c_{1j}') - \frac{\partial}{\partial \mathbf{r}}\left(\mathbf{D}_1 \frac{\partial}{\partial \mathbf{r}}\right)(\rho c_{1j}') &= -K_j(\psi_j' c_{1js}' - c_{2j}') + \rho Q_{1j} \\ \frac{\partial}{\partial t}(\varepsilon c_{2j}') + \frac{\partial}{\partial \mathbf{r}}(\varepsilon \mathbf{v} c_{2j}') - \frac{\partial}{\partial \mathbf{r}}\left(\mathbf{D}_{2j} \frac{\partial}{\partial \mathbf{r}}\right)(\varepsilon c_{2j}') &= K_j(\psi_j' c_{1js}' - c_{2j}') + \varepsilon Q_{2j} \\ \varepsilon = 1 - \rho, \quad K_j = nK_{0j}, \quad n = \rho / \sigma, \quad \sigma = \frac{4}{3}\pi a^3 \end{aligned} \quad (1.1)$$

Here ρ , ε , and \mathbf{w} , \mathbf{v} are the volume concentrations and velocities of the dispersed and continuous phases; n , σ , a are the countable concentration, volume, and radius of the drops of the dispersed phase; c_{1js}' is the concentration on the surface of the drops; K_{0j} is the coefficient of mass transfer of the j -th substance from the continuous medium referred to one drop; ψ_j' is the equilibrium coefficient of distribution of the substance between phases. The quantities Q_{1j} and Q_{2j} describe the formation of the j -th substance as a result of chemical reactions; they are referred to a unit volume of the corresponding phase. The tensor \mathbf{D}_1 characterizes the chaotic mixing of the drops, and tensor \mathbf{D}_{2j} characterizes such mixing and molecular diffusion of the j -th substance in the continuous phase. We will consider below $c_{1js}' = c_{1j}'$.

In the general case $c_{1js}' \neq c_{1j}'$, so that it is necessary to consider convective diffusion of substances within a drop, and so the convective diffusion in the medium surrounding this drop - otherwise it is impossible to formulate the boundary conditions on the surface of the drop. The latter complicates the problem extremely and, therefore, in extraction theory [1, 2] one usually takes $c_{1js}' = c_{1j}'$, changing appropriately the determination of the coefficient K_{0j} and ψ_j' . Here we will proceed in the same way, regarding ψ_j' , K_{0j} as well as Q_{1j} , Q_{2j} , \mathbf{D}_1 , \mathbf{D}_{2j} as certain functions of all concentrations, ρ , and other parameters.

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We note that Eqs. (1.1) represent a natural generalization of the equations of material balance used in simple calculations of extraction and absorption apparatuses [1, 2]. They were formulated in [3] for the quite important regime of extracting one substance.

Systems of equations (1.1) should be supplemented by equations of the hydromechanics of the two-phase system being considered. The equations of the conservation of mass of the phases are written in the form

$$\begin{aligned} \frac{\partial}{\partial t}(d_1\rho) + \frac{\partial}{\partial r}(\mathbf{w}d_1\rho) &= -\sum K_j(\psi_j'c_{1j}' - c_{2j}') \\ \frac{\partial}{\partial t}(d_2\varepsilon) + \frac{\partial}{\partial r}(\mathbf{v}d_2\varepsilon) &= \sum K_j(\psi_j'c_{1j}' - c_{2j}'), \quad c_{1js}' = c_{1j}' \end{aligned} \quad (1.2)$$

Here d_1 and d_2 are the densities of the materials of the dispersed and continuous phases which depend on their composition.

The equations of the conservation of momentum of the phases are written for simplicity with neglect of viscous stresses in both phases. We have [4]

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \mathbf{w} \frac{\partial}{\partial \mathbf{r}}\right)(d_1\rho\mathbf{w}) &= -\rho \frac{\partial p}{\partial \mathbf{r}} + d_1\rho\mathbf{g} + \mathbf{F} \\ \left(\frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}}\right)(d_2\varepsilon\mathbf{v}) &= -\varepsilon \frac{\partial p}{\partial \mathbf{r}} + d_2\varepsilon\mathbf{g} - \mathbf{F} \end{aligned} \quad (1.3)$$

where \mathbf{g} is the acceleration of gravity and \mathbf{F} is the force of interaction between the phases, referred to a unit volume of the mixture. In the expression for \mathbf{F} , which is assumed to be a known function of ρ and of the relative velocity $\mathbf{v} - \mathbf{w}$, the force acting on the drop from the average field of the pressure p in the continuous medium is not taken into account.

To close system (1.1)-(1.3) we must use the equations of state

$$d_1 = d_1(p, c_{1j}'), \quad d_2 = d_2(p, c_{2j}') \quad (1.4)$$

following from the thermodynamic analysis of the materials of the phases [1, 2].

Along with concentrations c_{1j}' and c_{2j}' it is convenient to consider the concentrations

$$c_{1j} = \rho c_{1j}', \quad c_{2j} = \varepsilon c_{2j}' \quad (1.5)$$

which are referred to a unit volume of the mixture.

The solution of the complete system of Eqs. (1.1)-(1.4) presents considerable difficulties. However, in many cases this system can be simplified appreciably. Some qualitative and quantitative conclusions which are quite important in the designing of absorption or extraction devices of various types can be made from an analysis of these simplified problems. We will examine below the simplest models of mass-transfer processes in apparatuses with a counter-flow and direct flow of the phases and with a horizontal flow of the continuous phase.

2. We will investigate simple extraction of a single substance in a countercurrent column under idealized mass-transfer conditions. Namely, we will consider that the transfer of the substance has practically no effect on the specific volumes of the phases and mass-transfer coefficient K , so that these quantities are constant over the height of the column. In addition, we neglect the concentration dependence of the distribution coefficient and local mixing in the column. These assumptions are not of fundamental importance; they all exhibit a typical apparatus character and are quite common in solvent extraction theory [1,2].

Considering a one-dimensional flow in a countercurrent column, we orient the x' axis in the direction of movement of the dispersed phase and select the origin at the place of its entrance into the column. Introducing dimensionless quantities and restricting ourselves to an investigation of steady extraction, we obtain from (1.1) the following simple equations for concentrations c_1 and c_2 from (1.5):

$$\begin{aligned} (1 - \gamma) \frac{dc_1}{dx} + k(\psi c_1 - c_2) &= 0, \quad \gamma \frac{dc_2}{dx} + k(\psi c_1 - c_2) = 0 \\ x = \frac{x'}{h}, \quad \gamma = \frac{v}{u}, \quad \psi = \frac{\varepsilon}{\rho} \psi', \quad k = \frac{Kh}{\varepsilon u}, \quad 0 \leq \gamma \leq 1, \quad k > 0 \end{aligned} \quad (2.1)$$

Here h is the column height; $v = \text{const}$ is the velocity of the counter-flow of the dispersion medium; $u = w - v$ is the velocity of motion of the drops in the stationary continuous medium. From (1.3) we have the equations for u and the gradient p at $\rho = \text{const}$

$$-\frac{dp}{dr} + dg = 0, \quad d = d_1\rho + d_2\varepsilon, \quad \rho \left(-\frac{dp}{dr} + d_1g \right) + F(\rho, u) = 0 \quad (2.2)$$

so that u can be considered a known function of ρ and of the physical parameters; it is clear that u is also constant. The equations of the conservation of mass (1.2) are satisfied identically in view of the assumption made concerning the constancy of the specific volume of the phases.

From (2.1) follows the relation

$$(1 - \gamma)c_1 - \gamma c_2 = \text{const} \quad (2.3)$$

which is used widely in practical calculations [1, 2].

For determinacy we will examine extraction from the dispersed phase, when the boundary conditions imposed on the solutions of system (2.1) have the form

$$c_1(0) = c_0 = \rho c_0', \quad c_2(1) = 0 \quad (2.4)$$

The solution of problem (2.1), (2.4) has the form

$$c_1 = \frac{c_0}{1 - \psi\gamma(1 - \gamma)^{-1}e^{-\lambda}} \left(1 - \frac{\psi\gamma}{1 - \gamma} e^{-\lambda(1-x)} \right) \quad (2.5)$$

$$c_2 = \frac{\psi c_0}{1 - \psi\gamma(1 - \gamma)^{-1}e^{-\lambda}} (1 - e^{-\lambda(1-x)}), \quad \lambda = k \frac{1 - (1 + \psi)\gamma}{\gamma(1 - \gamma)}$$

From the technological standpoint it is convenient to characterize the operation of the countercurrent reactor by the values of the coefficient of saturation of the dispersion medium l or the coefficient of extraction of the substance from the dispersed phase m , which are introduced by means of the relations [see (2.15)]

$$l = \frac{c_2(0)}{\psi c_0} = \frac{1 - e^{-\lambda}}{1 - \psi\gamma(1 - \gamma)^{-1}e^{-\lambda}}$$

$$m = 1 - \frac{c_1(1)}{c_1(0)} = 1 - \frac{1 - (1 + \psi)\gamma}{1 - (1 + \psi)e^{-\lambda}\gamma} \quad (2.6)$$

When $\gamma \rightarrow 0$ we have $l \rightarrow 1$, $m \rightarrow 0$, and when $\gamma \rightarrow 1$ we have on the contrary $l \rightarrow 0$, $m \rightarrow 1$. When $k \rightarrow \infty$, i.e., in the case of an unlimited increase of the column height h or coefficient K , the values of l and m approach the limits l_∞ , m_∞ , whereby

$$l_\infty = 1, \quad m_\infty = \frac{\psi\gamma}{1 - \gamma}, \quad \gamma < \gamma_0 = \frac{1}{1 + \psi} \quad (2.7)$$

$$l_\infty = \frac{1 - \gamma}{\psi\gamma}, \quad m_\infty = 1, \quad \gamma > \gamma_0$$

We emphasize that at certain values of γ , i.e., for certain relationships between the velocities of the phases in the reactor, l_∞ or m_∞ is less than unity, i.e., complete saturation or extraction is not achieved even with an unlimited increase of the column height.

Let us consider the operating characteristics of a column for fixed values of l or m , which are determined by practical requirements imposed on the real extraction process.

Thus, the requirement $l = \text{const}$ is characteristic for processes in which it is necessary to achieve a certain saturation of the dispersion medium by the substance being extracted from the dispersed phase. The operating characteristics of such processes, which represent the dependence of k on γ and ψ for given l , are obtained easily from the first relation (2.6). They have the form

$$k = \frac{\gamma(1 - \gamma)}{1 - (1 + \psi)\gamma} \ln \frac{1 - l}{1 - \psi\gamma(1 - \gamma)^{-1}l} \quad (l = \text{const}) \quad (2.8)$$

Similarly, the operating characteristics of processes satisfying the requirement $m = \text{const}$ are obtained from the second relation (2.6)

$$k = - \frac{\gamma(1-\gamma)}{1-(1+\psi)\gamma} \ln \frac{\psi\gamma - (1-\gamma)m}{\psi\gamma(1-m)} \quad (m = \text{const}) \quad (2.9)$$

The latter requirement is natural, for example, in processes of purifying the dispersed phase of some impurity.

It is easy to see that the quantities k from (2.8) and (2.9), which are regarded as functions of γ , are determined, respectively, in the regions

$$\begin{aligned} 0 \leq \gamma \leq \gamma_1(l, \psi) = (1 + \psi l)^{-1} & \quad (0 \leq l \leq 1) \\ m(m + \psi)^{-1} = \gamma_2(m, \psi) \leq \gamma \leq 1 & \quad (0 \leq m \leq 1) \end{aligned} \quad (2.10)$$

whereby, when $\gamma \rightarrow \gamma_1$ or $\gamma \rightarrow \gamma_2$, the quantity k becomes infinite.

The indicated behavior of the operating characteristics means simply that the extraction process with given $l = \text{const}$ or $m = \text{const}$ can by no means be accomplished at any value of γ from the interval $[0, 1]$. If this value lies outside the regions of (2.10), this process cannot be realized in principle even when $k \rightarrow \infty$. This same conclusion follows also from relations (2.7).

This is a fundamental conclusion which in a number of cases can be of decisive importance in the designing of extraction and absorption countercurrent reactors. In particular, from here it follows that some statements of the theory of solvent extraction which are taken for truth as self-evident are not only not evident but are not true in general. As an example we can cite the assumption of the continuous attainment of an equilibrium distribution of the substance between phases at the place of entry of one of the phases into a column of infinite height, the assumption of the asymptotic approach of the coefficient l and m to unity upon an increase of column height, and certain other statements [1, 2].

As $k \rightarrow \infty$, the quantities l and m approach the limits, which depend considerably on the relationships between the phase velocities γ , distribution coefficient ψ , etc. Apparently the indicated circumstances were not noticed in the theory [1, 2] because the investigation of fundamental equations of type (2.1) is generally replaced in this theory by examination of the first integral (2.3), and the constant figuring in it is usually expressed by a priori unknown concentrations of the substances in both phases at the outlet of these phases from the reactor.

Physically, the absence of a steady regime $l = \text{const}$ in the presence of a strong counter-flow ($\gamma > \gamma_1$) is related simply with the circumstance that the substance entering the reactor with the dispersed phase is not enough for proper saturation of a too large amount of the continuous medium. Likewise, the absence of the regime $m = \text{const}$ in the presence of a weak counter-flow means that the small amounts of the continuous medium are not enough for proper removal of the dispersed phase.

Let us now consider extraction under direct-flow conditions to which the ranges $\gamma < 0$ and $\gamma > 1$ of the change of γ correspond. If $\gamma < 0$, the dispersed phase overtakes the continuous phase and if $\gamma > 1$, vice versa. In the first case, equations (2.1) are valid as before, and in the second case the equations are obtained from (2.1) after transformation $x \rightarrow -x$. For direct flows the second condition of (2.4) loses meaning; we replace it by the condition $c_2(0) = 0$. The solution of the problem is written then in a form valid both for $\gamma < 0$ and for $\gamma > 1$

$$\begin{aligned} c_1 &= \frac{c_0}{(1 + \psi)|\gamma| - \text{sign } \gamma} (|\gamma| - \text{sign } \gamma + \psi|\gamma|e^{-\lambda x}) \\ c_2 &= \frac{\psi c_0 (|\gamma| - \text{sign } \gamma)}{(1 + \psi)|\gamma| - \text{sign } \gamma} (1 - e^{-\lambda x}), \quad \lambda = k \frac{(1 + \psi)|\gamma| - \text{sign } \gamma}{|\gamma| (|\gamma| - \text{sign } \gamma)} \end{aligned} \quad (2.11)$$

The coefficients of saturation l and extraction m are determined by the preceding formulas (2.6). We have from (2.6) and (2.11)

$$l = \frac{(|\gamma| - \text{sign } \gamma)(1 - e^{-\lambda})}{(1 + \psi)|\gamma| - \text{sign } \gamma}, \quad m = 1 - \frac{|\gamma| - \text{sign } \gamma + \psi|\gamma|e^{-\lambda}}{(1 + \psi)|\gamma| - \text{sign } \gamma} \quad (2.12)$$

The values of l_∞ and m_∞ corresponding to $k \rightarrow \infty$ are obtained from (2.12) for $\lambda \rightarrow \infty$. We see that the maximum possible values of l_∞ and m_∞ , equal to unity, are not attained no matter what the value of γ . (The exceptions are the limit values $\gamma \rightarrow -0$, when $l_\infty \rightarrow 1$, and $\gamma \rightarrow 1 + 0$, when $m_\infty \rightarrow 1$, which are of little interest.) Relations (2.12) permit continuing the operating characteristics (2.8) and (2.9) of the extraction processes constructed in the range $0 \leq \gamma \leq 1$ beyond the limits of this range.

3. As a second, somewhat opposite, example we will consider the dissolution (absorption) of a dispersed gas in a liquid under direct-flow and counter-flow conditions in a vertical direction. In this case the constant density of the gas d_1 will figure in the equation in place of the concentration c_1' , but in return the quantity ρ is variable.

We will assume for simplification that the density of the liquid d_2 and the distribution coefficient ψ' do not depend on the concentration $c = c_2'$; then in particular $\psi' d_1 = c_* = \text{const}$.

The two equations of the conservation of mass (1.2) and equation of material balance in the liquid from (1.1) are written in the form

$$\begin{aligned} d_1 \frac{d}{dx} [(w - \gamma) \rho] + k(c_* - c) &= 0, & w &= \frac{u}{u_0}, & \gamma &= \frac{v}{u_0} \\ d_2 \frac{d}{dx} (\epsilon \gamma) + k(c_* - c) &= 0 \\ \frac{d}{dx} (\epsilon \gamma c) + k(c_* - c) &= 0, & k &= \frac{Kh}{\epsilon_0 u_0}, & x &= \frac{x'}{h} \end{aligned} \quad (3.1)$$

Here u_0 , $\rho_0 = 1 - \epsilon_0$ is the relative velocity and volume concentration of the gas bubbles at the place of their entry into the column; the other notations are as before. The values of $\gamma < 0$ correspond to an ascending direct flow (the gas leads the liquid), the values of $0 \leq \gamma \leq 1$ to a counter-flow, and the values of $\gamma > 1$ to a descending direct flow (the gas lags behind the liquid).

The first two integrals of system (3.1) can be represented in the form

$$d_1 (w - \gamma) \rho - d_2 \epsilon \gamma = C_1, \quad (d_2 - c) \epsilon \gamma = C_2 \quad (3.2)$$

where C_1, C_2 are constants.

Obviously, $c \sim d_1 \ll d_2$ and, therefore, from the second relation (3.2) follows

$$\gamma = \gamma_0 \epsilon^{-1}, \quad \gamma_0 = v_0 u_0^{-1} \quad (3.3)$$

Here v_0 is the velocity of the liquid in the absence of gas. As before, the quantity u can be determined in terms of ρ from Eqs. (1.3) or (1.2), and the unknown c can be expressed in terms of ρ by means of the first integral of (3.2). Substituting these expressions and relations (3.3) into the first equations of (3.1), we obtain the equation for the only unknown ρ .

For an illustration of the possible dissolution regimes it suffices to consider only the equation describing the absorption of gas far from saturation. (Such dissolution is realized, for example, in cases when the dissolved gas enters into a rapidly occurring reaction.)

Assuming $c \ll c_*$, we have the equation

$$\frac{d}{dx} [(w - \gamma) \rho] + \frac{kc_*}{d_1} = 0, \quad \rho(0) = \rho_0 \quad (3.4)$$

The quantities u and K figuring in the determination of w , γ , and k should be expressed as functions of ρ . For determinacy we use here the relations obtained by Levich [5] for small and moderate Reynolds numbers R (up to $R \approx 700-800$)

$$u = b \frac{da^2}{\mu} f_u(\rho), \quad K_0 = b' \left(\frac{Ddg f_u(\rho)}{\mu} \right)^{1/2} f_k(\rho) a^{5/2}, \quad d \approx d_2 \epsilon \quad (3.5)$$

Here μ is the viscosity of the liquid; D is the coefficient of diffusion of the dissolved gas in the liquid; $f_u(\rho)$ and $f_k(\rho)$ are some correction functions which take into account the constrained character of the flow of the bubbles and diffusion from their surface. For small R we have $b = 2/9$, $b' = (8/3)(\pi/3)^{1/2}$; for large R , $b = 1/9$, $b' = (8/3)(\pi/2)^{1/2}$. We note that the calculations of certain other authors (see, for example, [1, 2]) also lead to formulas (3.5) for slightly different b and b' .

Neglecting the processes of fractionation and coalescence of the bubbles, we can write the equations

$$n = \rho / \sigma = \text{const}, \quad a = a_0 (\rho / \rho_0)^{1/2} \quad (3.6)$$

where a_0 is the initial radius of the bubbles. Then from (3.1), (3.5), and (3.6) we obtain the relations (we restrict ourselves to the case $\rho \ll 1$, when $f_u \approx f_k \approx 1$)

$$w = \left(\frac{\rho}{\rho_0}\right)^{2/3}; \quad k = 2\beta \left(\frac{\rho}{\rho_0}\right)^{5/6} \frac{\rho_0 d_1}{c_*}, \quad 2\beta = \frac{b' c_*}{\rho_0 d_1} \left(\frac{D d g}{\mu}\right)^{1/2} \frac{n h a_0^{5/2}}{\epsilon_0 u_0} \quad (3.7)$$

Introducing the unknown $\xi = \rho / \rho_0$ and using (3.7) and relation $\gamma \approx \gamma_0$ following from (3.3) for $\rho \ll 1$, we obtain from (3.4) the problem

$$-\frac{d}{dx} [\xi (\xi^{2/3} - \gamma_0)] + 2\beta \xi^{5/6} = 0, \quad \xi(0) = 1 \quad (3.8)$$

The solution of this problem is represented in the form

$$(1 - \xi^{5/6}) - 3\gamma_0 (1 - \xi^{1/6}) = \beta x \quad (3.9)$$

We will investigate relations (3.8) and (3.9) separately for values of γ_0 in different ranges.

Ascending Direct Flow ($\gamma_0 < 0$). The quantity $\xi(x)$ decreases monotonically from one for $x = 0$ to zero for $x = x_0$, where

$$x_0 = \beta^{-1} (1 - 3\gamma_0) > 0 \quad (3.10)$$

In the working section of the column ($0 \leq x \leq 1$) complete dissolution of the gas ($m = 1$) occurs if $x_0 < 1$. Otherwise the coefficient of extraction is equal ($x_0 > 1$) to

$$1 - m = \frac{\rho(u(\rho) - v_0)}{\rho_0(u_0 - v_0)} \Big|_{x=1} = \xi(1) \frac{\xi^{2/3}(1) - \gamma_0}{1 - \gamma_0}, \quad v \approx v_0 \quad (3.11)$$

Using (3.9) and (3.11), we can easily construct the operating characteristics of a column with an ascending direct flow. We note that the quantity x_0 , which determines the height at which the bubbles disappear completely, decreases rapidly with increase of β and decrease of $|\gamma_0|$.

Descending Direct Flow ($\gamma_0 > 1$). The relations characterizing absorption in this flow are obtained from (3.8) and (3.9) after changing the sign of x . Relation (3.10) for x_0 is replaced by the relation

$$x_0 = \beta^{-1} (3\gamma_0 - 1) \quad (3.12)$$

but expression (3.11) for m remains valid for the case being considered. We see that for the same $|\gamma_0|$ dissolution occurs more quickly in the ascending flow, when the dispersed gas leads the liquid. As follows from the results in Section 2, simple extraction, conversely, occurs slightly more quickly in the direct flow in which the continuous phase overtakes the dispersed.

Counter-Flow ($0 \leq \gamma_0 \leq 1$). As the gas dissolves the size of its bubbles and their relative velocity u decrease until at some level $x = x_m$ this velocity is comparable to the velocity of the counter-flow of liquid v_0 , i.e., until "flooding" occurs. We see from (3.8) that this level is determined by the condition $\xi(x_m) = \xi_m = \gamma_0^{3/2}$, i.e., from (3.9) we obtain the expression for x_m

$$x_m = \beta^{-1} [(1 - \gamma_0^{5/6}) - 3\gamma_0 (1 - \gamma_0^{1/6})] = \beta^{-1} (1 - 3\gamma_0 + 2\gamma_0^{5/6}) \quad (3.13)$$

The condition of the absence of flooding in the countercurrent column obviously has the form $x_m > 1$. In this case the coefficient of extraction is represented as before in the form (3.11). If x_m in (3.13) is less than unity, flooding occurs in the working part of the apparatus and it is necessary to investigate the return movement of small bubbles beginning at the level $x = x_m$. For this purpose it suffices to introduce a new coordinate $z = x_m - x$ and use the results obtained above for the descending direct flow. Introducing the new unknown $\zeta = \rho / \rho_m$, where ρ_m is the volume concentration of the rising bubbles for $x = x_m$ ($\rho_m = \xi_m \rho_0$), and the new parameters β_m, γ_m , which are expressed in terms of ρ_m, a_m just as β in (3.7) and γ_0 in (3.3) are expressed by ρ_0, a_0 , we obtain for ζ an equation of the same type as (3.9)

$$(1 - \zeta^{5/6}) - 3\gamma_m (1 - \zeta^{1/6}) = -2 + \zeta^{1/6} (3 - \zeta^{2/6}) = -\beta_m z \quad (3.14)$$

Complete dissolution of the gas bubbles in the column will occur if [compare with (3.12)]

$$z^\circ = \beta_m^{-1}(3\gamma_m - 1) = 2\beta_m^{-1} \leq x_m \quad (3.15)$$

The obvious equality $\gamma_m = 1$ is used in (3.14) and (3.15).

The volume concentration of gas emerging from the column along with the liquid is $\rho^\circ = \zeta(x_m)\rho_m$ (if, of course, $z^\circ > x_m$). In this case we have the following equation for the coefficient of extraction:

$$1 - m = \frac{\rho^\circ(v_0 - u(\rho^\circ))}{\rho_0(u_0 - v_0)} = \xi_m \zeta^\circ \frac{\gamma_0 - (\xi_m \zeta^\circ)^{2/3}}{1 - \gamma_0}, \quad \zeta^\circ = \zeta(\rho^\circ) \quad (3.16)$$

Using relation (3.14) and the definition of ρ° , we easily express m as a function of the parameters γ_0 , u_0 , k , and ρ_0 and construct further the working characteristics of the process.

Complete dissolution of the entire gas introduced into the column is evidently achieved when condition (3.15) is achieved. If the technological purpose consists in maximum absorption of the gas by the liquid in the countercurrent column, it is advantageous to select different parameters (h , ρ_0 , and others) such that the following relations are fulfilled:

$$x_m = 1 - \zeta, \quad \zeta \ll 1; \quad z^\circ \leq x_m$$

We note that the presence of flooding in the column and subsequent increase of small gas bubbles by the flow of the liquid leads to some equalization of the gas constant over the column height. Actually, the total volume concentration in the sections of the column in which there are both ascending and descending bubbles is given by the relation

$$\rho(x) = \rho_0 \xi(x) + \rho_m \zeta(x_m - x), \quad 0 \leq x \leq x_m, \quad 0 \leq x_m - x \leq z^\circ \quad (3.17)$$

In particular, at the height of flooding the concentration is equal not to ρ_m but to $2\rho_m$.

It is easy to see that the conclusions obtained do not change qualitatively also on rejecting the simplifying suggestions $\rho \ll 1$, $c \ll c_*$, etc. However, new steady regimes of absorption differing somewhat qualitatively from the regimes considered can appear near saturation ($c \sim c_*$).

4. We will consider briefly the processes of simple extraction and absorption of gas under conditions of a horizontal flow of the continuous medium (plate apparatuses, aeration and purification of industrial waters, etc.). We will use further the same simplifying assumptions as in Sections 2 and 3.

The equations of simple extraction, analogous to Eqs. (2.1), have the form

$$\gamma \frac{\partial c_1}{\partial y} + \frac{\partial c_1}{\partial x} + k(\psi c_1 - c_2) = 0, \quad \gamma \frac{\partial c_2}{\partial y} - k(\psi c_1 - c_2) = 0 \quad (4.1)$$

Here the y axis is directed along the velocity of the continuous medium, and the parameters γ , k , and ψ are determined by the previous relations (2.1) in which h is the height of the layer of the continuous medium. We write the boundary conditions in the form

$$c_1|_{x=0} = c_0, \quad c_2|_{y=0} = 0 \quad (4.2)$$

The equations of the characteristics of system (4.1), which determine the trajectories of the drops of the dispersed phase, have the form $z = y - \gamma x = \text{const}$. It is natural to introduce new coordinates x , z in which problem (4.1), (4.2) takes the form

$$\begin{aligned} \frac{\partial c_1}{\partial x} + \gamma(\psi c_1 - c_2) &= 0, & c_1|_{x=0} &= c_0 \\ -\frac{\partial c_2}{\partial z} - \gamma(\psi c_1 - c_2) &= 0 & c_2|_{z=0} &= 0 \end{aligned} \quad (4.3)$$

We apply to (4.3) the Laplace transform with respect to variable z and denote the transforms of functions c_1 and c_2 in terms of φ_1 and φ_2 . We obtain the problem

$$\frac{d\varphi_1}{dx} + \gamma(\psi\varphi_1 - \varphi_2) = 0, \quad \varphi_2 = \frac{\psi\gamma}{p + \gamma} \varphi_1, \quad \varphi_1|_{x=0} = \frac{c_0}{p} \quad (4.4)$$

The solution of this problem has the form

$$\varphi_1 = \frac{c_0}{p} \exp \frac{-\psi \gamma p x}{p + \gamma}, \quad \varphi_2 = \frac{\psi \gamma c_0}{p + \gamma} \exp \frac{-\psi \gamma p x}{p + \gamma} \quad (4.5)$$

where p is the Laplace variable.

Using the known properties of the Laplace transform and the table values of the originals, after certain transformations we obtain from (4.5) the relations

$$c_1(x, z) = c_0 e^{-\psi \gamma x} \left[1 + 2\gamma \sqrt{\psi x} \int_0^{\sqrt{z}} e^{-\gamma t^2} I_1(2\gamma \sqrt{\psi x} t) dt \right] \quad (4.6)$$

$$c_2(x, z) = 2c_0 \psi \gamma e^{-\psi \gamma x} \int_0^{\sqrt{z}} e^{-\gamma t^2} I_0(2\gamma \sqrt{\psi x} t) dt, \quad z = y - \gamma x$$

The solutions (4.6) of problem (4.3) are determined, obviously, in the range

$$0 \leq x \leq 1, \quad z > 0 \quad (4.7)$$

When $z < 0$ it is necessary to take $c_1 \equiv c_2 \equiv 0$ (the dispersed phase is completely absent in this case).

Of practical interest are the apparatuses whose length L greatly exceeds the height of the layer h . Setting $z = L/h$, $z_r \approx y_r \approx r$, we obtain the following formal relations for the coefficients of saturation and extraction in the reactor of the type being considered:

$$l \approx \frac{1}{\psi c_0} \int_0^1 c_2(x, r) dx, \quad m \approx 1 - \frac{1}{c_0 r} \int_0^r c_1(1, z) dz \quad (4.8)$$

Relation (4.8) can also be used in constructing the operating characteristics of the extraction process.

Absorption of gas when $\rho \ll 1$, $c \ll c_*$ does not depend on the position of the point along the flow of liquid, and the equation corresponding to (3.8) acquires the form

$$-\frac{d}{dx} \xi^{3/2} + 2\beta \xi^{3/2} = 0, \quad \xi|_{x=0} = 1 \quad (4.9)$$

Hence we have

$$\xi = (1 - \beta x)^{2/3} \quad (4.10)$$

If $x_0 = \beta^{-1} > 0$, where β is defined in (3.7), then part of the gas emerges from the liquid layer without being dissolved in it, and

$$1 - m = \frac{\rho u(\rho)}{\rho_0 u_0} \Big|_{x=1} = \xi^{2/3}(1) \quad (4.11)$$

If $x_0 < 1$, all gas introduced is dissolved ($m = 1$), and there is no gas in the range $x_0 \leq x \leq 1$. The trajectories of the bubbles are determined by the relations [see (4.10)]

$$\frac{dx}{dy} = \frac{w}{\gamma_0} = \frac{\xi^{2/3}}{\gamma_0} = \frac{(1 - \beta x)^{2/3}}{\gamma_0}, \quad y = y_0 + \frac{5\gamma_0}{\beta} [1 - (1 - \beta x)^{5/3}] \quad (4.12)$$

where y_0 is the coordinate of the point of entry of the bubble into the liquid layer.

We note that results completely analogous to those obtained above can be obtained easily also for any other theoretical or empirical relations for u and K_0 differing from (3.5). The relations presented permit optimizing various types of reactors with respect to various indices and selecting, for example, the optimal value of γ for fixed ρ , a , and h , determining the optimal dispersity (i.e., the quantity a) as a function of the other parameters, etc.

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